An Overview: Peak-to-Average Power Ratio Reduction Techniques for OFDM Signals

Aparna Tiwari, Karuna Markam
ayonija12@gmail.com, karuna_markam@rediffmail.com
Department of Electronics & Communication Engineering
Madhav Institute of Technology & Science, Gwalior, M.P. India

Abstract—Orthogonal Frequency Division Multiplexing (OFDM) has become the popular modulation technique in high speed wireless communications. It is more advantageous over other technologies. But despite its advantages it has some obstacles also. The high peak-to-average ratio is the main obstacle which causes non-linearity at the receiving end. In this paper, we review and analyze different OFDM PAPR reduction techniques, based on computational complexity, bandwidth expansion, spectral spilling and performance. We also discuss some methods of PAPR reduction for multiuser OFDM broadband communication systems.

Keywords: Complementary cumulative distribution function (CCDF), High power amplifier (HPA), Multiuser OFDM, OFDM, Peak-to-average power ratio (PAPR)

1. INTRODUCTION

New techniques for digital transmission have developed to meet the increasing demand for higher data rates in communications which can be used in both wired and wireless environments. To meet out the high spectral efficiency and high data rate, an efficient modulation scheme is to be employed. Orthogonal Frequency Division Multiplexing (OFDM), which is one of multi-carrier modulation (MCM) techniques, offers a considerable high spectral efficiency, multipath delay spread tolerance, immunity to the frequency selective fading channels and power efficiency [1], [2]. As a result, OFDM has been chosen for high data rate communications and has been widely deployed in many wireless communication standards such as Digital Video Broadcasting (DVB) and based mobile worldwide interoperability for microwave access (mobile WiMAX) based on OFDM access technology [3]. OFDM have several attractive features which make it more advantageous for high speed data transmission over other data transmission techniques. These features includes [5, 6]

i. High Spectral Efficiency
ii. Robustness to channel fading
iii. Immunity to impulse interferences
iv. Flexibility
v. Easy equalization

But inspite of these benefits there are some obstacles in using OFDM:

a. OFDM signal exhibits very high Peak to Average Power Ratio (PAPR)
b. Very sensitive to frequency errors (Tx. & Rx.offset)
c. Inter Carrier Interference (ICI) between the Subcarriers

The high peak-to-average ratio is the main obstacle. Therefore, it is important and necessary to research on the characteristics of the PAPR including its distribution and reduction in OFDM systems, in order to utilize the technical features of the OFDM.

As one of characteristics of the PAPR, the distribution of PAPR, which bears stochastic characteristics in OFDM systems, often can be expressed in terms of Complementary Cumulative Distribution Function (CCDF). Recently, some researchers have reported on determination of the PAPR distribution based on different theoretics and hypotheses [4]–[10]. Moreover, various approaches also have been proposed to reduce the PAPR including clipping [11]–[14], nonlinear companding transforms [24]–[29], Partial Transmission Sequence (PTS) and Selective Mapping (SLM)[35]. These schemes can mainly be categorized into signal scrambling techniques, such as block codes and PTS etc., and signal distortion techniques such as clipping.

Although some techniques of PAPR reduction have been summarized but it is still indeed needed to give a comprehensive review including some motivations of PAPR reductions, such as power saving, and to compare some typical methods of PAPR reduction through theoretical analysis and simulation results directly. An effective PAPR reduction technique should be given the best trade off between the capacity of PAPR reduction and transmission power, data rate loss, implementation complexity and Bit Error-Ratio (BER) performance etc. In this paper, firstly we investigate the distribution of PAPR based on the characteristics of the OFDM signals. Then, we analyze four typical techniques of PAPR reduction and propose the criteria of PAPR reduction in OFDM systems in details.

What is PAPR?

The PAPR is the relation between the maximum power of a sample in a given OFDM transmit symbol divided by the average power of that OFDM symbol. PAPR occurs when in a multicarrier system the different sub-carriers are out of phase with each other. At each instant they are different with respect to each other at different phase values. When all the points achieve the maximum value simultaneously; this will cause the output envelope to suddenly shoot up which causes a ‘peak’ in the output envelope. Due to presence of large number of independently modulated subcarriers in an OFDM system, the peak value of the system can be very high as compared to the average of the whole system. This ratio of
the peak to average power value is termed as Peak-to-Average Power Ratio.

2. DEFINITION OF PAPR

An OFDM signal consists of a number of independently modulated sub-carriers which can give a large PAPR when added up coherently. When N signals are added with the same phase they produce a peak power that is N times the average power of the signal. So OFDM signal has a very large PAPR, which is very sensitive to nonlinearity of the high power amplifier. In OFDM, a block of N symbols \{X, k = 0,1,....,N-1\} k, is formed with each symbol modulating one of a set of subcarriers, \{f, k = 0,1,....,N -1\} k. The N subcarriers are chosen to be orthogonal, that is, \( f_k f_k = D \), where \( Df = 1 \ NT \) and \( T \) is the original time period. The resulting signal is given as:

\[
x(t) = \sum_{n=0}^{N-1} X_n e^{j2\pi f_k t},
\]

PAPR is defined as:

\[
PAPR = \frac{\max|X(t)|^2}{E[|X(t)|^2]},
\]

where \( E[\cdot] \) denotes the expectation operator.

**PAPR Problem**

One of the new problems emerging in OFDM systems is the so-called Peak to Average Power Ratio (PAPR) problem. The input symbol stream of the IFFT should possess a uniform power spectrum, but the output of the IFFT may result in a non-uniform or spiky power spectrum. Most of transmission energy would be allocated for a few instead of the majority subcarriers. This problem can be quantified as the PAPR measure. It causes many problems in the OFDM system at the transmitting end.

**Effect of PAPR**

There are some obstacles in using OFDM in transmission system in contrast to its advantages:

i. A major obstacle is that the OFDM signal exhibits a very high Peak to Average Power Ratio (PAPR).

ii. Therefore, RF power amplifiers should be operated in a very large linear region. Otherwise, the signal peaks get into non-linear region of the power amplifier causing signal distortion. This signal distortion introduces inter-modulation among the subcarriers and out of band radiation. Thus, the power amplifiers should be operated with large power back-offs. On the other hand, this leads to very inefficient amplification and expensive transmitters. Thus, it is highly desirable to reduce the PAPR.

iii. These large peaks cause saturation in power amplifiers, leading to inter-modulation products among the subcarriers and disturbing out of band energy. Therefore, it is desirable to reduce the PAPR.

iv. To reduce the PAPR, several techniques have been proposed such as clipping, coding, peak windowing, Tone Reservation and Tone Injection. But, most of these methods are unable to achieve simultaneously a large reduction in PAPR with low complexity, with low coding overhead, without performance degradation and without transmitter receiver symbol handshake.

v. Complexity is increased in the analog to digital and digital to analog converter.

**Criteria for PAPR Reduction Method Selection**

The criterion of the PAPR reduction is to find the approach that it can reduce PAPR largely and at the same time it can keep the good performance in terms of the following factors as possible. The following criteria should be considered in using the techniques:

a. The high capability of PAPR reduction is primary factor to be considered in selecting the PAPR reduction technique with as few harmful side effects such as in-band distortion and out-of-band radiation.

b. Low average power: Although it also can reduce PAPR through average power of the original signals increase, it requires a larger linear operation region in HPA and thus resulting in the degradation of BER performance.

c. Low implementation complexity: Generally, complexity techniques exhibit better ability of PAPR reduction. However, in practice, both time and hardware requirements for the PAPR reduction should be minimal.

d. No bandwidth expansion: The bandwidth expansion directly results in the data code rate loss due to side information. Moreover, when the side information are received in error unless some ways of protection such as channel coding employed. Therefore, when channel coding is used, the loss in data rate is increased further due to side information. Therefore, the loss in bandwidth due to side information should be avoided or at least be kept minimal.

e. No BER performance degradation: The aim of PAPR reduction is to obtain better system performance including BER than that of the original OFDM system. Therefore, all the methods, which have an increase in BER at the receiver, should be paid more attention in practice.

f. Without additional power needed: The design of a wireless system should always take into consideration the efficiency of power.

3. PAPR REDUCTION TECHNIQUES IN OFDM SYSTEMS

In this section, we mainly discuss five typical techniques for PAPR reduction in OFDM systems.

**A. Clipping and Filtering**

The simplest and most widely used technique of PAPR reduction is to basically clip the parts of the signals that are outside the allowed region [11]. For example, using HPA with saturation level below the signal span will automatically cause the signal to be clipped. For amplitude clipping, that is...
\[ C(x) = \begin{cases} x, & x \leq A \\ A, & x > A \end{cases} \quad (18) \]

where \( A \) is preset clipping level and it is a positive real number. Generally, clipping is performed at the transmitter. However, the receiver needs to estimate the clipping that has occurred and to compensate the received OFDM symbol accordingly. Typically, at most one clipping occurs per OFDM symbol, and thus the receiver has to estimate two parameters: location and size of the clip. However, it is difficult to get this information. Therefore, clipping method introduces both in band distortion and out of band radiation into OFDM signals, which degrades the system performance including BER and spectral efficiency. Filtering can reduce out of band radiation after clipping although it cannot reduce in-band distortion. However, clipping may cause some peak re-growth so that the signal after clipping and filtering will exceed the clipping level at some points. To reduce peak re-growth, a repeated clipping-and-filtering operation can be used to obtain a desirable PAPR at a cost of computational complexity increase. As improved clipping methods, peak windowing schemes attempt to minimize the out of band radiation by using narrowband windows such as Gaussian window to attenuate peak signals.

### B. PTS and SLM

In a typical OFDM system with PTS approach to reduce the PAPR, the input data block in is partitioned into disjoint subblocks, which are represented by the vectors

\[ X^{(m)} = [X_0^{(m)}, X_1^{(m)}, \ldots X_{N-1}^{(m)}] \quad (35). \]

Therefore, we can get

\[ X = \sum_{m=0}^{M-1} X^{(m)} \]

Where \( X^{(m)} = [X_0^{(m)}, X_1^{(m)}, \ldots X_{N-1}^{(m)}] \) with \( X_k^{(m)} \) or 0 (0 \( \leq \) m \( \leq \) M-1). In general, for PTS scheme, the known sub-block partitioning methods can be classified into three categories [35]: adjacent partition, interleaved partition and pseudo-random partition.

![Figure 1: Block diagram of PTS technique](image)

Then, the sub-blocks are transformed into time-domain partial transmit sequences

\[ x^{(m)} = [x_0^{(m)}, x_1^{(m)}, \ldots x_{LN-1}^{(m)}] = IFFTLN X^{(m)} \]

These partial sequences are independently rotated by phase factors

\[ b = \{b_m = e^{j\theta_m}, m = 0, 1, \ldots, M-1\} \]

The objective is to optimally combine the M sub-blocks to obtain the time domain OFDM signals with the lowest PAPR

\[ \hat{x} = \sum_{m=0}^{M-1} b_m x^{(m)} \]

Therefore, there are two important issues should be solved in PTS: high computational complexity for searching the optimal phase factors and the overhead of the optimal phase factors as side information needed to be transmitted to receiver for the correct decoding of the transmitted bit sequence.

Suppose that there are \( W \) phase angles to be allowed, thus \( b_m \) can has the possibility of \( W \) different values. Therefore, there are \( W^M \) alternative representations for an OFDM symbol. To reduce the searching complexity and avoid/reduce the usage of side information, many extensions of PTS have been developed recently [39]. In [66], authors proposed a novel scheme, which is based on a nonlinear optimization approach named as simulated annealing, to search the optimal combination \( b_m \) of phase factors with low complexity. In general, PTS needs MIPFFT operations for each data block, and the number of the required side information bits is \( M \log_2 W \), where \( x \) denotes the smallest integer that does not exceed \( x \). Similarly, in SLM, the input data sequences are multiplied by each of the phase sequences to generate alternative input symbol sequences. Each of these alternative input data sequences is made the IFFT operation, and then the one with the lowest PAPR is selected for transmission [40]. A block diagram of the SLM technique is depicted in Fig. 2. Each data block is multiplied by different phase factors, each of length\( L \), resulting in different data blocks. Then, the \( v \)th phase sequence after multiplied is \( X^v = [X_0 b_v, X_1 b_v, \ldots X_{N-1} b_v] \) \((v=0, 1, \ldots, V-1)\). Therefore, OFDM signals becomes as \( X^v = [X_0 b_v, X_1 b_v, \ldots X_{N-1} b_v] \).
Therefore, OFDM signals becomes as

\[ x^v(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n b_{v,n} e^{2\pi j f_c t} \]

where \( 0 \leq t \leq NT \), \( v = 1, 2, \ldots, V-1 \)

Among the data blocks, \( X^v \) \((v = 0, 1, \ldots, V-1)\) only one with the lowest PAPR is selected for transmission and the corresponding selected phase factors \( b_{v,n} \) also should be transmitted to receiver as side information is \( \log_2V \). For implementation of SLM OFDM systems, the SLM technique needs V IFFT operation and the number of required bits as side information is \( \log_2V \) for each data block. Therefore, the ability of PAPR reduction in SLM depends on the number of phase factors and the design of the phase factors. Some extensions of SLM also have been proposed to reduce the computational complexity and number of the bits for side information transmission [36].

C. Nonlinear Companding Transforms

One of the most attractive schemes is nonlinear companding transform due to its good system performance including PAPR reduction and BER, low implementation complexity and no bandwidth expansion.

The first nonlinear companding transform is the \( \mu \)-law companding which is based on the speech processing algorithm \( \mu \)-law, and it has shown better performance than that of clipping method [24]. \( \mu \)-law mainly focuses on enlarging signals with small amplitude and keeping peak signals unchanged, and thus it increase the average power of the transmitted signals and possibly results in exceeding the saturation region of HPA to make the system performance worse. In fact, the nonlinear companding transform is also an especial clipping scheme.

The differences between the clipping and nonlinear companding transform can be summarized as:

i. Clipping method deliberately clips large signals when the amplitude of the original OFDM signals is larger than the given threshold, and thus the clipped signals cannot be recovered at the receiver. However, nonlinear companding transforms compand original OFDM signals using the strict monotone increasing function. Therefore, the companded signals at the transmitter can be recovered correctly through the corresponding inversion of the nonlinear transform function at the receiver;

ii. Nonlinear companding transforms enlarge the small signals while compressing the large signals to increase the immunity of small signals from noise, whereas clipping method does not change the small signals. Therefore, clipping method suffers from three major problems: in-band distortion, out-of-band radiation and peak re-growth after digital analog conversion. As a result, the system performance degradation due to the clipping may not be optimistic.

Nonlinear companding transform is a type of nonlinear process that may lead to significant distortion and performance loss by companding noise. Companding noise can be defined that the noises are caused by the peak re-growth after DAC to generate in-band distortion and out-band noise, by the excessive channel noises magnified after inverse nonlinear companding transform etc. For out-of-band noise, it needs to be filtered and oversampled. For in-band distortion and channel noises magnified, they need to iterative estimation. Unlike Additive White Gaussian Noise (AWGN), companding noise is generated by a process known and that can be recreated at the receiver, and subsequently be removed. In [28], the framework of an iterative receiver has been proposed to eliminate commanding noise for companded and filtered OFDM system.

4. PAPR REDUCTION FOR MULTIUSER OFDM SYSTEMS

Recently, multiuser OFDM also has received much attention due to its applicability to high speed wireless multiple access communication systems. In multiuser OFDM system,

<table>
<thead>
<tr>
<th>Power Increase</th>
<th>Implementation Complexity</th>
<th>Bandwidth Expansion</th>
<th>BER Degradation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clipping</td>
<td>No</td>
<td>Low</td>
<td>No</td>
</tr>
<tr>
<td>Coding</td>
<td>No</td>
<td>Low</td>
<td>Yes</td>
</tr>
<tr>
<td>PTS/SLM</td>
<td>No</td>
<td>High</td>
<td>Yes</td>
</tr>
<tr>
<td>NCT</td>
<td>No</td>
<td>Low</td>
<td>No</td>
</tr>
<tr>
<td>TR/TI</td>
<td>Yes</td>
<td>High</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Table 1: Comparison of different PAPR Technologies

Data streams from multiple users are orthogonally multiplexed onto the downlink and uplink sub channels. In a multiuser OFDM system, a group of carriers is assigned for each user with adaptive modulation, bit and power allocation. Obviously, the characteristics including distribution of the PAPR for each user in uplink multiuser OFDM is the same as that of the PAPR in single user OFDM system since the data of each user will be transmitted to channels independently in uplink multiuser OFDM system. Therefore, the PAPR can be reduced according to these schemes mentioned above in the uplink multiuser OFDM systems. However, the characteristics of the PAPR in downlink multiuser OFDM is different from that of the PAPR in single user OFDM system since the data composed from different users will be transmitted to channels successively in downlink multiuser OFDM system. Therefore, the PAPR reduction is more complicated in a downlink than that in OFDM uplink in multiuser OFDM systems. If downlink PAPR reduction is achieved by some approaches which have been designed for OFDM, each user has to process the whole data frame and then demodulate the assigned subcarriers to extract their own information. Thus, it introduces additional processing for each user at the receiver.
Therefore, we mainly describe some modifications of PAPR reduction techniques for the downlink multiuser OFDM systems.

a. TS/SLM for PAPR reduction in multiuser OFDM systems: PTS and SLM techniques can easily be modified for PAPR reduction in downlink of multiuser OFDM systems. For PTS, subcarriers assigned to one user are grouped into one or more sub-blocks, and then PTS can be applied to sub-blocks for all users. As side information, the selected phase factor for each sub-block can be embedded into the pre-reserved subcarrier in each sub-block. Note that, the pre-reserved subcarrier does not undergo the phase rotation in each sub-block. Similarly, some of the subcarriers can be used to transmit side information when the modified SLM is applied to reduce the PAPR for multiuser OFDM systems. All users use the information carried by these subcarriers to obtain the phase sequence is used at the transmitter, and thus the data for each user can be recovered correctly.

b. TR for PAPR reduction in multiuser OFDM systems: In the TR technique for multiuser OFDM systems, the symbols in peak reduction subcarriers are optimized for the whole data frame in both amplitude and phase. At the same time, some peak reduction subcarriers are assigned to each user in the TR for PAPR reduction.

5. CONCLUSIONS

OFDM is a very attractive technique for wireless communications due to its spectrum efficiency and channel robustness. One of the serious drawbacks of OFDM systems is that the composite transmit signal can exhibit a very high PAPR when the input sequences are highly correlated. In this paper, we described several important aspects, as well as provide a mathematical analysis, including the distribution of the PAPR, in OFDM systems. Four typical techniques to reduce PAPR have been analyzed, all of which have the potential to provide substantial reduction in PAPR at the cost of less in data rate, transmit signal power increase, BER performance degradation, computational complexity increase, and so on. We also showed that it is possible to reduce the PAPR of for multiuser OFDM systems.

REFERENCES


